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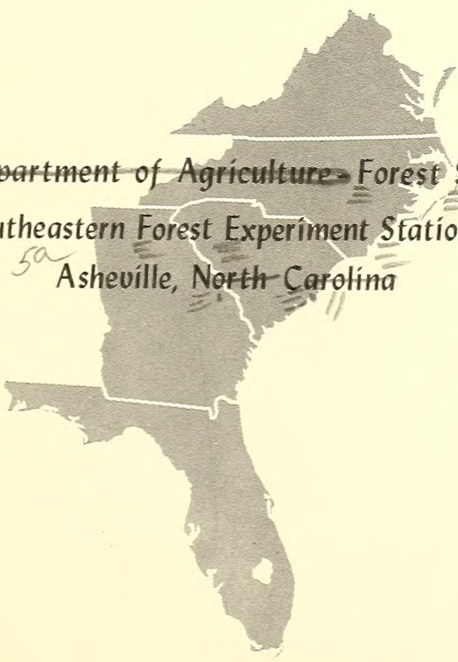

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BOARD-FOOT AND CUBIC-FOOT VOLUME COMPUTING EQUATIONS FOR SOUTHEASTERN TREE SPECIES

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INTRODUCTION

Wide acceptance of Bitterlich's (2) method of sampling, popularized in this country by Grosenbaugh (3), with adaptations such as the variable plot used by Forest Survey in the Southeast, has opened a new era in forest surveying. The efficiency of these sampling methods, accompanied by the timely availability of electronic computing machines, has made it feasible to collect and retain large amounts of forest inventory information.

Each tree on the sample plot now can be treated individually, recording such descriptive data as species, tree quality, and crown class, along with stem dimensions, amount of cull, and rate of growth. This information can be transferred to individual punch cards with area information such as forest type, stand size, site quality, and stand condition. The method has greatly expanded the ability of the forest manager to analyze the situation in large forest holdings and that of the forest resource analyst to do so for a state or region. Still lacking, however, are volume equations that can provide precise volume estimates for individual trees to match this fund of detailed tree and area data.

The Forest Survey organizations of the Intermountain and Southeastern Forest Experiment Stations have used volume estimating equations for several years. Bennett, McGee, and Clutter (1), Langdon (4), and many others have used volume equations in the construction of volume tables. Most common have been adaptations of the D^2H equations described by Spurr (5), in which d. b. h. and merchantable length are the measured variables. Such equations prepared for uniform stands of one species, such as conifer plantations, may provide accurate estimates for individual trees. But exploratory work done by the authors with several variations of the D^2H equation failed, as a general rule, to provide the desired precision in natural stands of the Southeast. This was especially true of tests on hardwoods.

This paper presents equations that use one or more additional measured variables in an attempt to estimate individual tree volumes more accurately.

SOURCE OF TREE MEASUREMENT DATA

Nearly 10 years ago, a system was set up by Forest Survey at the Southeastern Station for defining and coding tree sections and recording their dimensions, gross volume, cull, log grade, utilization for timber products, etc. The system has been followed in two cull and log-grade studies and one utilization study in Virginia, two cull and log-grade studies and two utilization studies in North Carolina, one cull and log-grade study in South Carolina, one utilization study in Florida, and a utilization and volume-table study in Georgia. These mensurational studies varied in size from about 300 to 1,000 trees on 56 to 108 sample areas.

Individual tree volumes were obtained by the accumulation of section volumes for each sample tree.

EXPLORATORY WORK WITH EQUATION FORM

Tree measurement data for one softwood and one hardwood (loblolly pine and sweetgum) were studied in experiments with equation form. Multiple regression techniques were used to analyze the following board-foot and cubic-foot volume equations:

$$V_B = a+b_1(D^2H)+b_2(S^2H)+b_3(DSH)+b_4(H)+b_5\left(\frac{H}{D-S}\right)+b_6(D^2)+b_7\left(\frac{1}{D-8.9}\right)$$

$$V_C = a+b_1(D^2H_c)+b_2(D^2)+b_3(H_c)+b_4\left(\frac{H_c}{D}\right)$$

where:

V_B = Gross board-foot volume (International $\frac{1}{4}$-inch rule)

V_C = Gross cubic-foot volume to 4.0 inches outside bark (excluding bark)

D = Diameter at breast height or 1.5 feet above bottleneck for normally swell-butted trees 1/

S = Diameter inside bark at top of saw-log portion

H = Length of saw-log portion in feet

H_c = Length of cordwood section to 4.0 inches outside bark

The coefficients of multiple correlation derived for all possible combinations of variables were compared as a preliminary analytical procedure. Table 1 presents a summary of this test.

1/ For bottomland species such as cypress and tupelo with normal butt-swell extending above 3 feet, diameter should be measured 1.5 feet above noticeable swell or "bottleneck."

Table 1. --Coefficients of multiple correlation from initial tests on equation form

Species	Type of equation	Best R^2 without S	Best R^2 with S
Loblolly pine	Board-foot	96.8	98.5
	Cubic-foot	98.6	--
Sweetgum	Board-foot	95.5	98.2
	Cubic-foot	70.3	--

These tests and others involving range of accuracy, contribution of additional variables, and empirical accuracy of prediction indicated that an equation of the form $V = a+b(D^2H)$ might be acceptable for estimating both board-foot and cubic-foot volume of certain conifers. For other more variable species, particularly hardwoods, an additional stem dimension is required to compensate for differences in stem form that very commonly occur in trees of the same d. b. h. and merchantable length.

Different equation forms and choices of variables by species and tree size might be justified for specialized forest inventories. It is believed, however, that for extensive inventories such as those conducted by Forest Survey, it is best to settle on one or two equation forms. This is important from the standpoint of (1) training cruisers and maintaining accuracy in their work, and (2) programming and checking machine computations.

Diameter inside bark (d. i. b.) at a selected point on the stem was chosen as a third dimension (fig. 1). In the case of sawtimber trees, this point coinsides with the saw-log top. In pole trees and culls, it is the point of noticeable change in stem taper where such a point can be distinguished. When such a point cannot be distinguished, it is the midpoint of the stem. This third stem dimension was selected for the following reasons:

1. The upper limit of the saw-log portion commonly coincides with a point of noticeable change in stem taper. The more-or-less cylindrical saw-log portion often changes there to a cone-shaped upper stem.
2. D. i. b. at the saw-log top is one of the dimensions commonly considered by cruisers in establishing the limit of minimum saw-log merchantability.

ACCURACY OF D. I. B. ESTIMATES

The thought of using d. i. b. as a variable in volume equations immediately suggests the need for tests of the average timber cruiser's ability to estimate it. It appeared possible that d. i. b. estimates might be too inaccurate to be of value in refining volume equations. Consequently, two trial tests were made.

Preliminary Field Test

Eighty-five trees of various species, diameters, and heights were measured beforehand and marked to identify the point of d. i. b. measurement. A group of 11 Forest Survey cruisers were asked to estimate d. o. b. and d. i. b. at the point marked on each tree. The men differed in experience from a few months to over 3 years, but none had been trained in making d. i. b. estimates.

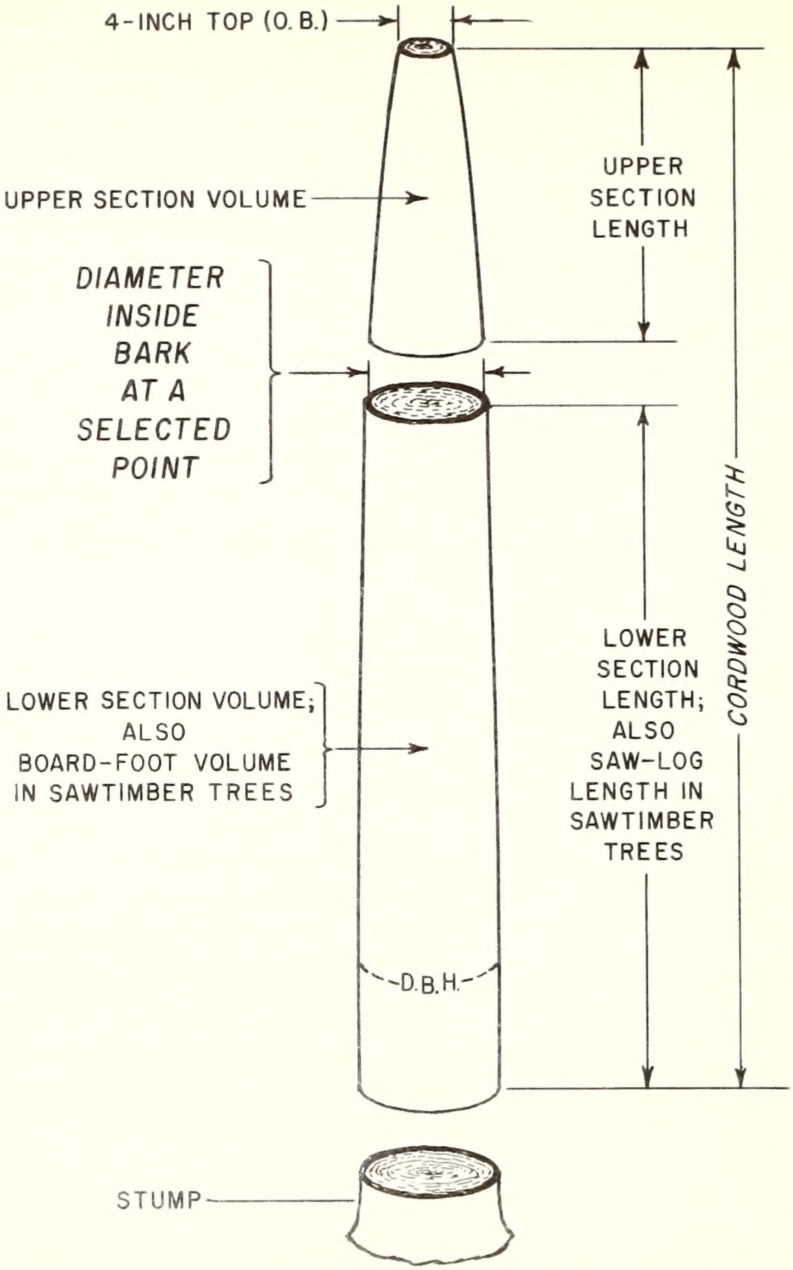

Figure 1. --The diameter inside bark at a selected point, when used with d. b. h., cordwood length, and saw-log length, adds to the precision of volume estimates.

The average error was 0.75 inch for d. o. b. and 0.76 inch for d. i. b. Only one estimator's error in d. i. b. averaged more than 0.90 inch. It was interesting to note that d. i. b. was estimated with approximately the same accuracy as d. o. b.

The preliminary tests indicated that d. i. b. would add to the precision of volume estimates except in the case of cubic-foot volume in small pole-timber trees.

A Test For Maximum Error

Although the preliminary test indicated that timber cruisers can estimate d. i. b. with acceptable accuracy, there was doubt as to the reliability of such a test in evaluating accuracy under normal field conditions. A second test was set up under conditions expected to provide the least accurate estimates that might be expected of Forest Survey crews. No special training or detailed instructions were given. All crews were simply given field plot sheets with an added space for top d. i. b. and were asked to begin recording it for all trees 5.0 inches d. b. h. or larger. The crews did not know that their estimates might be checked.

After the estimates had progressed for one month, 118 trees on a subsample of 16 plots were climbed to check the d. i. b. estimates. Results of this test showed an average error of 0.47 inch. The 6 men who had also taken part in the earlier test averaged 0.22 inch less error in the second test. Evidently some training has been gained from the earlier experience.

With field training and occasional checks, it is believed that cruisers should maintain an accuracy of d. i. b. estimates somewhat better than shown in the second test.

FINAL VOLUME ESTIMATING EQUATIONS

The final volume equations take the following form:

BOARD FEET $V_B = a + b_1 DSH + b_2 H$

CUBIC FEET $V_C = b_1 DSH + b_2 SU + b_3 (SU)^2$

where:

V_B = Gross board-foot volume (International $\frac{1}{4}$-inch rule)

D = Diameter at breast height or 1.5 feet above point of bottleneck for normally swell-butted trees.

S = Scaling diameter (d. i. b.) at the top of the lower section (also top of saw-log portion in sawtimber trees)

H = Length of lower section (also saw-log portion in sawtimber trees)

and

V_C = Gross cubic-foot volume to the 4.0-inch top outside bark, excluding bark volume

U = Upper section length in feet. This is the section from the top of the lower section to the 4.0-inch o. b. top.

The board-foot equation was weighted by the reciprocal of DSH. The cubic-foot equation was handled in two parts separating volume in the lower section from volume in the upper section. The former, $V = b_1DSH$, was weighted by the reciprocal of DSH. The latter, $V = b_2SU+b_3(SU)^2$, was weighted by the reciprocal of SU.

The coefficients for gross board-foot (International $\frac{1}{4}$-inch rule) volume equations of the form $V_B = a+b_1DSH+b_2H$ for the principal southeastern species are listed in table 2. The equation coefficients for gross cubic-foot volume (excluding bark) to a 4.0-inch top outside bark are listed in table 3.

The cubic-foot equation $V_C = b_1DSH+b_2SU+b_3(SU)^2$ can be used in two segments. The first portion, b_1DSH, supplies the volume of the lower section in all size trees. The remainder, $b_2SU+b_3(SU)^2$, computes the volume of the upper section in all size trees.

Table 2. --Board-foot volume equation coefficients for principal species of the Southeast

Species	Number of trees used	a	b_1	b_2
Longleaf pine	101	-11.89	0.035955	-0.0686
Slash pine	87	-09.40	.035909	-0.2762
Loblolly pine	323	-13.21	.038112	-0.2644
Shortleaf pine	65	-11.07	.035355	-0.0773
Pond pine	49	-10.07	.035584	-0.2131
Virginia pine	156	-03.05	.037057	-0.3735
White pine	126	-05.97	.038338	-0.4564
Baldcypress	87	-17.12	.037542	-0.1716
Pondcypress	69	-06.79	.033052	-0.2804
Blackgum	98	-12.86	.037147	-0.3387
Yellow-poplar	83	-14.93	.041232	-0.5470
Sweetgum	133	-07.52	.040328	-0.7234
Red maple	67	-00.42	.040385	-1.0743
White oak	77	-04.02	.039088	-0.6716
Chestnut oak	109	-08.61	.039357	-0.4924
Northern red oak	59	-04.60	.040752	-0.8123
Scarlet oak	117	-05.44	.039133	-0.6653
Hickory	72	-05.26	.038061	-0.4626
Ash	45	-02.44	.038834	-0.7236
Birch	47	-04.54	.038733	-0.5999

Table 3. --Cubic-foot volume equation coefficients for principal species of the Southeast

Species	Number of trees used	b_1	b_2	b_3
Longleaf pine	191	0.005127	0.01951	0.00002740
Slash pine	203	.004866	.01712	.00003942
Loblolly pine	363	.005106	.01941	.00004870
Shortleaf pine	114	.005045	.01551	.00006159
Pond pine	58	.004956	.01667	.00005722
Virginia pine	236	.005287	.01927	.00004910
White pine	130	.005337	.02028	.00003668
Baldcypress	100	.005157	.02275	.00002719
Pondcypress	95	.004951	.01830	.00003344
Blackgum	141	.005056	.01617	.00006025
Yellow-poplar	125	.005307	.01949	.00002839
Sweetgum	139	.005285	.02244	.00002658
Red maple	71	.005407	.02291	.00001495
White oak	92	.005312	.01907	.00002843
Chestnut oak	112	.005127	.02191	.00002040
Northern red oak	60	.005331	.01970	.00002550
Scarlet oak	125	.005224	.01907	.00002712
Hickory	73	.005218	.02099	.00002196
Ash	56	.005304	.01861	.00003298
Birch	50	.005383	.02168	.00001814

TESTS OF ACCURACY

Statistical Tests of the Volume Regression

The reliability of the board-foot and cubic-foot volume equations was tested for six commercially important species of the Southeast. Included were longleaf, shortleaf, and loblolly pines, blackgum, yellow-poplar, and white oak. Analyses of error were made at the 5-percent level. The half-widths of the confidence intervals for tree of mean size are given in table 4.

In addition, the equations were tested and found to be consistently reliable over the full range of tree diameters for each species.

Tests With Independent Samples of Trees

The most meaningful test of a volume equation is, of course, its performance in actual use. For that reason, several of the equations were checked to determine how well they estimated the volumes of measured trees other than those used in deriving them.

Table 4. --Statistical accuracy of volume equations for a few selected species

Species	Type of equation	Trees used in regression	Half width of confidence interval for tree of mean size 1/
		Number	Percent
Longleaf pine	Board-foot	101	1.72
	Cubic-foot	191	1.39
Loblolly pine	Board-foot	323	1.10
	Cubic-foot	363	.97
Shortleaf pine	Board-foot	65	4.46
	Cubic-foot	114	2.47
Blackgum	Board-foot	98	1.83
	Cubic-foot	141	2.80
Yellow-poplar	Board-foot	83	1.48
	Cubic-foot	125	2.06
White oak	Board-foot	77	1.61
	Cubic-foot	92	2.21

1/ This figure may be interpreted to mean that there are only 5 chances in 100 that the true volume of a tree of average size will differ from the predicted volume by more than the percent shown.

Trees used in these tests were measured on cutting operations in the Piedmont and mountains of Georgia entirely outside the area from which the sample tree measurements for the equations were obtained. The equations were found to be consistently accurate for trees of varying lengths over a range of diameters from 5 to 21 inches. Methods described by Spurr (5) were used to compute aggregate deviation and average deviation values, which are presented in table 5.

Table 5. --Tests of volume equations with independent samples of measured trees

Species	Trees used--		Gross volume		Error	
	constructing the equation	in the test	Scaled	Computed	Aggregate deviation	Average deviation
	- - - Number - - -		- - - Volume - - -		- - - Percent - - -	
BOARD-FOOT EQUATIONS						
Loblolly pine	323	96	8,715	8,828	1.28	6.34
Shortleaf pine	65	54	4,142	3,956	-4.70	7.43
Blackgum	98	21	3,007	3,034	.89	6.23
Yellow-poplar	83	12	3,282	3,429	4.29	5.34
CUBIC-FOOT EQUATIONS						
Loblolly pine	363	189	2,110.7	2,198.6	4.00	6.19
Shortleaf pine	114	106	1,027.3	998.3	2.90	6.01
Blackgum	141	25	696.5	697.0	.07	5.75
Yellow-poplar	125	13	604.6	619.2	2.36	4.17

APPLICATION OF VOLUME EQUATIONS

The above tests indicate that satisfactory estimates of tree volume can be obtained with these volume equations. However, two restrictions must be observed if the equations are to be used most effectively:

1. The Merchantability Standards Observed by the Cruiser Should Match Those on Which the Equations are Based.

Table 6. --Minimum saw-log top by d. b. h. and broad species group
(In inches)

D. b. h.	Softwoods	Hardwoods
9	5.5	--
10	6.0	--
11	6.6	8.0
12	7.0	8.0
13	7.4	8.0
14	7.8	8.0
15	8.2	8.2
16	8.6	8.6
17	9.0	9.0
18	9.4	9.4
19	9.7	9.7
20	10.0	10.0
21	10.3	10.3
22	10.6	10.6
23	10.8	10.8
24	11.0	11.0
25	11.2	11.2
26	11.4	11.4
30+	12.0	12.0

The merchantability standards used in taking sample tree measurements were designed to coincide with general cutting practices in the Southeast. Softwoods 9.0 inches or larger and hardwoods 11.0 inches or larger having at least one 8-foot log and less than two-thirds cull in the saw-log portion were measured for board-foot volume. The upper limit of the saw-log portion was generally taken to a variable minimum top. However, excessive roughness or rot were limits to merchantability in some instances. Table 6 lists the variable minimum tops by d. b. h. and broad species group. Cubic-foot volume measurements were made on all sample trees 5.0 inches d. b. h. and larger. Merchantable cubic-foot volume was taken from a 1-foot stump to 4.0 inches outside bark.

2. Accurate Tree Volumes Depend on Accurate Tree Measurements

To take full advantage of the precision offered by the equations, trees should be measured as exactly as possible.

D. b. h. always should be measured to tenths of inches. For trees used in developing the regressions, d. b. h. was measured with a diameter tape whenever possible. In other cases two calipered diameters were averaged.

Lower section length and length to 4.0-inch top outside bark should be recorded in feet (the difference between these is upper section length). Professional cruisers should frequently check tree lengths with a hypsometer. It is recommended that they measure at least one or more trees in each clump and all extremely tall trees as a minimum. Those who estimate timber only occasionally should measure the lengths of all trees.

Cruisers should train themselves to estimate top d. i. b., making frequent checks unless an instrument is used to make this measurement (outside bark). Actual measurements following ocular estimates serve as the best training check. This can be done by climbing and measuring estimated trees

or by estimating standing trees on cutting operations and making check measurements as they are felled. Another method almost as effective is to stand off at varying distances from trees to estimate d. i. b. four or five feet above ground; then measure actual d. o. b. and bark thickness at that point.

Bark thickness within species usually is closely correlated with diameter of stem, although appearance of the bark's surface provides additional clues to its thickness. Knowledge of bark thickness can be gained by measuring it at points within reach on a range of tree sizes.

Timber cruisers should be expected to maintain an accuracy of ± 5 percent in nine out of ten estimates of d. i. b. after training and using the above mentioned methods of checking themselves.

LITERATURE CITED

(1) Bennett, F. A., McGee, C. E., and Clutter, J. L.
1959. YIELD OF OLD-FIELD SLASH PINE PLANTATIONS. U. S. Forest Serv. Southeast. Forest Expt. Sta. Paper 107, 19 pp.

(2) Bitterlich, W.
1948. DIE WINKELZÄHLPROBE. Allgemeine Forst-und Holzwirtschaftliche Zeitung 59(1/2): 4-5.

(3) Grosenbaugh, L. R.
1952. PLOTLESS TIMBER ESTIMATES--NEW, FAST, EASY. Jour. Forestry 50: 32-37, illus.

(4) Langdon, O. G.
1958. VOLUME TABLES FOR SOUTH FLORIDA SLASH PINE. U. S. Forest Serv. Southeast. Forest Expt. Sta. Res. Notes 120, 2 pp.

(5) Spurr, S. H.
1952. FOREST INVENTORY. Ronald Press. 476 pp. New York

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